SOME IMPORTANT MATHEMATICAL FORMULAE

Number System

- **Natural Number** - counting numbers - 1,2,3,4,5………
- **Whole Number** - Zero + counting numbers - 0,1,2,3,4……
- **Integer** - Whole number + negative of counting number - …..-3, -2, -1, 0, 1, 2, 3……..
- **Even Number** - Divisible by 2 - 2,4,6,8….
- **Odd Number** - Not divisible by 2 - 1,3,5,7….
- **Prime Number** - Divisible by 1 and itself only - 2,3,5,7,11…
- **Composite Number** - Not prime – 4,6,8,9,10....
- **Co-prime Number** – Not having any common factor – (2,3),(4,7),(10,13)…
- **Rational Number** – Expressed as $\frac{p}{q}$, q≠0 - $\frac{1}{2}$,3,-4,$\frac{7}{16}$….
- **Irrational Number** – Not Expressed as $\frac{p}{q}$, q≠0 – $\sqrt{2}$,$\sqrt{3}$,$\sqrt{5}$….
- Face and Place values of Digits

  e.g. In 893546 , **face value** of 3 = 3 , **place value** of 3 = 3 × 1000 = 3000

- Dividend(D) = Divisor(d) × Quotient(q) + Remainder(r) $\frac{d\,)\,D\,(\,q}{r}$
- Sum of first n natural numbers = $1 + 2 + 3 + \cdots + \mathrm{n} = \Sigma n = \frac{n(n+1)}{2}$
- Sum of square of first n natural numbers = $1^2 + 2^2 + 3^2 + \cdots + \mathrm{n}^2 = \Sigma\mathrm{n}^2 = \frac{n(n+1)(2\mathrm{n}+1)}{6}$
- Sum of cubes of first n natural numbers = $1^3 + 2^3 + 3^3 + \cdots + \mathrm{n}^3 = \Sigma\mathrm{n}^3 = \left[\frac{n(n+1)}{2}\right]^2$

HCF and LCM

- Product of two numbers = HCF × LCM
- HCF of fractions = $\frac{HCF\ of\ numerators}{LCM\ of\ denominators}$
- LCM of fractions = $\frac{LCM\ of\ numerators}{HCF\ of\ denominators}$

Squares and Cubes

Number(n)	Square(n^2)	Cube(n^3)
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000
11	121	1331
12	144	1728
13	169	2197
14	196	2744
15	225	3375
16	256	4096
17	289	4913
18	324	5832
19	361	6859
20	400	8000
21	**441**	**9261**
22	484	10648
23	529	12167
24	576	13824
25	625	15625
26	676	17576
27	729	19683
28	784	21952
29	841	24389
30	900	27000

Surds and Indices

1. $a^0 = 1$
2. $a^m \times a^n = a^{m+n}$
3. $a^m + a^n = \frac{a^m}{a^n} = a^{m-n}$
4. $(a^m)^n = a^{mn}$
5. $(ab)^m = a^m b^m$
6. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
7. $a^{-m} = \frac{1}{a^m}$

1. $\sqrt[n]{ab} = \sqrt[n]{a}.\sqrt[n]{b}$
2. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
3. $(\sqrt[n]{a})^n = (a^{1/n})^n = a$
4. $(\sqrt[m]{a})^n = \sqrt[m]{a^n}$
5. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

Algebra

1. $a^2 - b^2 = (a + b)(a - b)$
2. $(a + b)^2 = a^2 + b^2 + 2ab$
3. $(a - b)^2 = a^2 + b^2 - 2ab$
4. $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$
5. $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$
6. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
7. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
8. $(a + b)^2 - (a - b)^2 = 4ab$
9. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$
10. $(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab - 2ac + 2bc$
11. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - ac + bc)$
12. If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

'VBODMAS' Rule = V- Vinculum or Bar ('–') ; B- Bracket, O- of ; D- Division ; M- Multiplication ; A- Addition ; S- Subtraction

Percentage

- Convert decimal fraction into percentage e.g. 0.035 = 0.035 × 100% = 3.5%
- Convert percentage into decimal fraction e.g. 150% = $\frac{150}{100}$ = 1.5
- x% of y = $\frac{x \times y}{100}$
- If x_1 is increased to x_2, then percentage increase = $\frac{x_2 - x_1}{x_1}$ × 100%
- If x_1 is decreased to x_2, then percentage increase = $\frac{x_1 - x_2}{x_1}$ × 100%
- If an amount is first increased by x% and then reduced by x%, then percentage change will be a decrease of $\frac{x^2}{100}$ %
- When the value of an object is first changed by x% and then by y%, then

  Net effect = $\left[\pm x \pm\ y + \frac{(\pm x)(\pm y)}{100}\right]$ %

- If value of A is x% more (or less) than the value of B, then value of B is $\left(\frac{x}{100 \pm x} \times 100\right)$% less (or more) than the value of A.
- If the population of a town is P and annual rate of increase (or decrease) is r%, then,
  Population after n years = $P\left(1 \pm \frac{r}{100}\right)^n$

  Population before n years = $\frac{P}{\left(1 \pm \frac{r}{100}\right)^n}$

- On increasing (or decreasing) the cost of certain article by x% a person can buy 'a' kg less (or more) article in Rs y, then increase cost of the article = Rs $\frac{xy}{100 \times a}$
- Initial cost = $\frac{xy}{(100 \pm x) \times a}$

Ratio and Proportion

- a : b = $\frac{a}{b}$
- a : b :: c : d → $\frac{a}{b} = \frac{c}{d}$ → ad = bc
- Mean proportion of a and b is $\sqrt{ab}$

Mixture & Alligation

$$\frac{Quantity\ of\ cheaper\ article}{Quantity\ of\ costly\ article} = \frac{Cost\ price\ of\ a\ unit\ of\ a\ costly\ article - Mean\ price}{Mean\ price - Cost\ price\ of\ a\ unit\ of\ a\ cheaper\ article}$$

Profit, Loss and Discount

Profit = S.P. – C.P.

Loss = C.P. – S.P.

Profit% = $\frac{Profit}{CP} \times 100\%$

Loss% = $\frac{Loss}{CP} \times 100\%$

SP = $\frac{CP \times (100 + P\%)}{100}$

SP = $\frac{CP \times (100 - L\%)}{100}$

CP = $\frac{SP \times 100}{100 + P\%}$

CP = $\frac{SP \times 100}{100 - L\%}$

Discount % = $\frac{MP - SP}{MP} \times 100$

MP = $\frac{SP \times 100}{100 - D\%}$

- If by selling any two articles at the same price, there is a profit of r% on one and a loss of r% on the other article. There is always a loss of $\left(\frac{r^2}{100}\right)$ % in the transaction.
- If CP of x articles is equal to SP of y articles, then profit % = $\frac{x-y}{y} \times 100$ (if x > y) and loss % = $\frac{y-x}{y} \times 100$ (if x< y).
- A seller sells his goods on CP but uses y gm weight in the place of x gm weight, then profit % = $\frac{x-y}{y} \times 100\%$
- Single equivalent discount of two discounts $r_1\% \; and \; r_2\% = \left[r_1 + r_2 - \frac{r_1 \times r_2}{100}\right]\%$

Average

1. Average = $\frac{Sum\ of\ numbers}{Number\ of\ numbers} = \frac{x_1 + x_2 + x_3 + \ldots\ldots + x_n}{n}$
2. The average of first n natural numbers is $\left(\frac{n+1}{2}\right)$
3. The average of consecutive natural even numbers upto n is $\left(\frac{n}{2} + 1\right)$
4. The average of first n natural even number is (n + 1)
5. The average of consecutive natural odd numbers upto n is $\left(\frac{n+1}{2}\right)$
6. The average of first n natural odd numbers is n
7. The average of first n multiples of x is $\frac{x(n+1)}{2}$

Simple & Compound Interest

- Simple Interest (SI) = $\frac{P \times r \times t}{100}$
- Amount = $P\left(1+\frac{rt}{100}\right)$ = SI + P
- Compound Interest = $P\left[\left(1+\frac{r}{100}\right)^{t}-1\right]$
- Amount = $P\left(1+\frac{r}{100}\right)^{t}$
- The difference between compound interest and simple interest obtained on Rs P at r% per annum for 2 year is $P\left(\frac{r}{100}\right)^{2}$ and for 3 year is $\frac{Pr^{2}(300+r)}{(100)^{3}}$

Time and Work

1) If A, B, and C can do a work in x, y, and z days respectively, then they will complete the work in $\frac{xyz}{xy+yz+zx}$ days by working together.
2) If m_1 person complete w_1 work in d_1 days by working h_1 hours per day earning r_1 and m_2 persons complete w_2 work in d_2 days by working h_2 hours per day earning r_2, then $\frac{m_1 \times d_1 \times h_1}{w_1 r_1} = \frac{m_2 \times d_2 \times h_2}{w_2 r_2}$
3) If A can do a work in x days and B can do y% fast then B will complete the work in $\frac{100x}{(100+y)}$ days.

Speed, Time and Distance

1) Speed = (Distance)/Time
2) When two bodies/objects A and B are moving with speed x km/h and y km/h respectively, then the relative speed of two bodies/object is as follows
   a) (x – y) km/h, if they are moving in the same direction.
   b) (x + y) km/h, if they are moving in the opposite direction.
3) Average speed = (Total distance covered by a body)/(Total time taken by a body)
4) To convert the speed of km/h into m/s, we simply multiply by 5/18 in the given value and also to convert the speed of m/s into km/h, we simply multiply by 18/5 in the given value.
5) If two trains start at the same time from two points A and B towards each other and after crossing they take a and b hours in reaching B and A respectively. Then, A's speed: B's speed = √(b):√a.

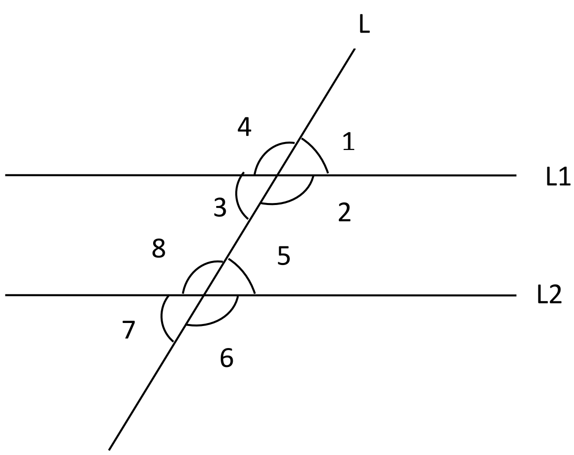
Boat and Stream

- Downstream = boat running along the stream.
- Upstream = boat running against the stream.
- If the speed of a boat in still water is *u* km/h and speed of the stream is *v* km/h, then
- Speed downstream = ($u + v$) km/h
- Speed upstream = (u – v) km/h
- If the speeds of downstream and upstream be x and y km/h respectively, then

Speed in still water = $\frac{x+y}{2}$ km/h

Speed of stream = $\frac{x-y}{2}$ km/h

Geometry



Angle 1 = 5 = 3 =7

Angle 2 = 4 = 6 = 8

- Acute angle = Angle between 0° and 90°
- Obtuse angle = Angle between 90° and 180°
- Reflex angle = Angle greater than 180° and less than 360°
- Right angle = Angle equal to 90°
- Straight angle = Angle equal to 180°
- Complementary angle = Two angles whose sum is 90°
- Supplementary angle = Two angles whose sum is 180°

Triangles

- Equilateral Triangle = All sides of a triangle is equal
- Isosceles Triangle = Two sides of a triangle is equal
- Scalene Triangle = Triangle having three sides of different lengths

- Congruent Triangle = If every angle and side of a triangle is equal to the corresponding angle and side of another triangle.
- Sum of all the angles of a triangle is 180°
- The angle made by increasing a side of a triangle is known as external angle which is equal to the sum of two opposite internal angles.

Quadrilaterals

- Sum of all the angles of a quadrilateral is 360°
- A quadrilateral in which both pairs of opposite sides are parallel and equal, is known as parallelogram.
- A parallelogram having all the sides equal is known as rhombus.
- Diagonals of rhombus intersect each other at 90°.
- A quadrilateral in which one pair of opposite sides are parallel is known as trapezium.

Mensuration

2D Shapes

Name	Area	Perimeter	Diagonal
Triangle	A=$\frac{1}{2}\times$ base $\times$ height =$\sqrt{s(s-a)(s-b)(s-c)}$ Where s = $\frac{a+b+c}{2}$	a + b + c	-
Isosceles triangle	A = $\frac{b}{4}\sqrt{4a^2-b^2}$	2a + b	-
Equilateral triangle	A = $\frac{\sqrt{3}}{4}a^2$	3a	-
Parallelogram	A = b × h	2(a + b)	-
Rectangle	A = L × B	2(L + B)	$\sqrt{L^2+B^2}$
Square	A = a^2	P = 4a, P = 2d$\sqrt{2}$	$\sqrt{2}$a
Rhombus	A = $\frac{1}{2}\times d_1\times d_2$	P = 4a, P = $2\sqrt{d_1^2+d_2^2}$	-
Trapezium	A = $\frac{1}{2}$(a + b) × h	a + b + AD +BC	-
Circle	A = πr^2	Circumference = 2πr	-

3D Shapes

Name	Volume	Lateral Surface Area/ Curved Surface Area	Total Surface Area/ Length of diagonal
Cuboid	V = l × b × h	LSA = 2h(l + b)	TSA = 2(lb + bh + hl) D = $\sqrt{h^2 + b^2 + l^2}$
Cube	V = a^3	LSA = $4a^2$	TSA = $6a^2$ Length of diagonal = a$\sqrt{3}$
Cylinder	V = $\pi r^2 h$	CSA = 2πrh	TSA = 2πr(r + h)
Cone	V = $\frac{1}{3}\pi r^2 h$	CSA = πrl Slant height(l) =$\sqrt{h^2 + r^2}$	TSA = πr(r + l)
Frustum of Cone	V = $\frac{\pi h}{3}(r^2 + R^2 + rR)$	CSA = π(R + r)l	TSA = π[(r + R)l + $r^2 + R^2$]
Sphere	V = $\frac{4}{3}\pi r^3$	-	SA = $4\pi r^2$

Trigonometry

- 180° = π radians
- 1° = $\frac{\pi}{180}$
- sin θ = $\frac{P}{H}$ — sin θ = 1/cosec θ
- cos θ = $\frac{B}{H}$ — cos θ = 1/sec θ
- tan θ = $\frac{P}{B}$ — tan θ = 1/cot θ
- cot θ = $\frac{B}{P}$ — cot θ = 1/tan θ
- sec θ = $\frac{H}{B}$ — sec θ = 1/cos θ
- cosec θ = $\frac{H}{P}$ — cosec θ = 1/sin θ

Trigonometric Ratios of Some Angles

Angle (θ)	0°	30°	45°	60°	90°
Sinθ	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1
Cosθ	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0
Tanθ	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞
Cotθ	∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0
Secθ	1	$2/\sqrt{3}$	$\sqrt{2}$	2	∞
Cosecθ	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1

1) sin(90°– θ) = cos θ
2) cos(90°– θ) = sin θ
3) tan(90°– θ) = cot θ
4) cot(90°– θ) = tan θ
5) sec(90°– θ) = cosec θ
6) cosec(90°– θ) = sec θ
7) sin(x + y) = sin(x)cos(y) + cos(x)sin(y)
8) cos(x + y) = cos(x)cos(y) – sin(x)sin(y)
9) tan(x + y) = (tan x + tan y)/ (1– tan x. tan y)
10) sin(x – y) = sin(x)cos(y) – cos(x)sin(y)
11) cos(x – y) = cos(x)cos(y) + sin(x)sin(y)
12) tan(x – y) = (tan x–tan y)/ (1+tan x. tan y)
13) sin(2x) = 2sin(x) cos(x) = [2tan x/(1+$\tan^2$ x)]
14) cos(2x) = $\cos^2$(x) – $\sin^2$(x) = [(1-$\tan^2$ x)/(1+$\tan^2$ x)]
15) cos(2x) = 2$\cos^2$(x) – 1 = 1 – 2$\sin^2$(x)
16) tan(2x) = [2tan(x)]/ [1 – $\tan^2$(x)]
17) sec (2x) = $\sec^2$ x/(2 - $\sec^2$ x)
18) cosec (2x) = (sec x. cosec x)/2
19) sin x · cos y = $\frac{\sin(x+y)+\sin(x-y)}{2}$
20) cos x · cos y = $\frac{\cos(x+y)+\cos(x-y)}{2}$
21) sin x · sin y = $\frac{\cos(x+y)-\cos(x-y)}{2}$
22) sin x + sin y = $2sin\frac{x+y}{2}cos\frac{x-y}{2}$
23) sin x – sin y = $2cos\frac{x+y}{2}sin\frac{x-y}{2}$
24) cos x + cos y = $2cos\frac{x+y}{2}cos\frac{x-y}{2}$
25) cos x – cos y = $-2sin\frac{x+y}{2}sin\frac{x-y}{2}$

Pie Char

Value of a particular sector = $\frac{\text{Percent value of the sector}}{100} \times$ Total value

= $\frac{\text{Degree value of the sector}}{360°} \times$ Total value

Permutation and Combination Formula

$nP_r = \frac{n!}{(n-r)!}$

$nC_r = nC_{n-r} = \frac{n!}{(n-r)! \times r!}$

The formula in Statistics

- **Mean** = $\frac{\text{Sum of all the elements}}{\text{Number of elements}}$
- **Mode** = The value that is most frequent in the dataset or maximum times of repeated numbers.
- **Median**

  If n is odd, then M = $\frac{n+1}{2}$th term

  If n is even, then M = $\frac{\frac{n}{2}\text{th term}+\left(\frac{n}{2}+1\right)\text{th term}}{2}$

- **Variance** = $\sigma^2 = \frac{\sum(x-\bar{x})^2}{n}$
- **Standard Deviation** = S = σ

Arithmetic Progression

- n^{th} term $T_n = a + (n - 1)d$

  Where, a= 1^{st} term and d= common difference

- Sum of n^{th}term $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[a + l]$

  Where l = last term

Geometric Progression

- $\mathrm{n^{th}\ term\ T_n}$ = ar^{n-1} where r= common ratio
- S_n= $\frac{a(r^n-1)}{r-1}$ for r > 1
- S_∞ = $\frac{a}{1-r}$ for r < 1

Analytical Geometry

- Distance between the two points $(x_{1,}y_1)$ and (x_2, y_2) in the plane is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ OR $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$.
- **Section formula** $\left(\frac{\mathrm{mx_2+nx_1}}{\mathrm{m+n}}, \frac{\mathrm{my_2+ny_1}}{\mathrm{m+n}}\right)$ for internal division,
  $\left(\frac{\mathrm{mx_2-nx_1}}{\mathrm{m-n}}, \frac{\mathrm{my_2-ny_1}}{\mathrm{m-n}}\right)$ for external division.
- **Mid point formula** $\left(\frac{\mathrm{x_1+x_2}}{2}, \frac{\mathrm{y_1+y_2}}{2}\right)$.
- **Centriod formula** $\left(\frac{\mathrm{x_1+x_2+x_3}}{3}, \frac{\mathrm{y_1+\ y_2+y_3}}{3}\right)$